

Appendix

1. The elements k_{ij} and K_0 of dynamic-stiffness matrix of layer $\mathbf{S}_{P1-P2-SV}^L$.

$$A_1 = ts_1s_2(D_1 - D_2); \quad A_2 = s_1(D_1 - D_3); \quad A_3 = s_2(D_2 - D_3); \quad A_4 = ts_1(E_2 - 2G^*);$$

$$A_5 = ts_2(E_1 - 2G^*); \quad A_6 = (E_1 - E_2)(D_3 - 1); \quad B_1 = s_1(t^2 - 1)D_1; \quad B_2 = s_2(t^2 - 1)D_2;$$

$$B_3 = s_1s_2(t^2 - 2D_3 + 1); \quad B_4 = P_1ts_2(D_2 - 1); \quad B_5 = P_2ts_1(D_1 - 1); \quad B_6 = (P_1 - P_2)(D_3 - 1);$$

$$SS = \sin(ks_1d) \sin(ks_2d); \quad SC = \sin(ks_1d) \cos(ks_2d); \quad CS = \cos(ks_1d) \sin(ks_2d);$$

$$CC = \cos(ks_1d) \cos(ks_2d);$$

$$K_0 = 2A_1[(A_3CS - A_2SC) \cos ktd + A_2 \sin ks_1d - A_3 \sin ks_2d] + [2A_2A_3(CC - 1) + (A_3^2 + A_2^2 + A_1^2)SS] \sin ktd;$$

$$k_{11} = k_{44} = k(t + 1/t)G^* A_1 [A_1SS \cos ktd + (A_2SC - A_3CS) \sin ktd];$$

$$k_{12} = -k_{45} = -kG^* [(r_1 + r_2SS - r_1CC) \sin ktd + (r_3SC + r_4CS) \cos ktd - r_3 \sin ks_1d - r_4 \sin ks_2d];$$

$$r_1 = (D_1 + D_2 - 2D_3)B_3 - A_2B_2 - A_3B_1; \quad r_2 = A_2B_1 + A_3B_2 - 2A_1^2 - B_3(A_2/s_2 + A_3/s_1);$$

$$r_3 = A_1(2A_2 - B_1 + B_3/s_2); \quad r_4 = A_1(B_2 - 2A_3 - B_3/s_1);$$

$$k_{13} = -k_{46} = n^{-1}k(1 + t^2)G^* [(r_1 + r_2SS - r_1CC) \sin ktd + (r_3SC + r_4CS) \cos ktd - r_3 \sin ks_1d - r_4 \sin ks_2d];$$

$$r_1 = -s_2A_2 - s_1A_3; \quad r_2 = s_1A_2 + s_2A_3; \quad r_3 = -s_1A_1; \quad r_4 = s_2A_1;$$

$$k_{14} = k_{41} = k(t + 1/t)G^* [A_1(A_3 \sin ks_2d - A_2 \sin ks_1d) \sin ktd - A_1^2 \sin ks_1d \sin ks_2d];$$

$$k_{15} = k(t^2 + 1)A_1G^* [r_1(\cos ks_1d - \cos ks_2d) \sin ktd + (r_2 \sin ks_1d + r_3 \sin ks_2d) \cos ktd - r_2SC - r_3CS];$$

$$r_1 = (1 - D_3)/t; \quad r_2 = -s_1(D_1 - 1); \quad r_3 = s_2(D_2 - 1);$$

$$k_{16} = \frac{k(t^2 + 1)A_1G^*}{n} \left[\frac{1}{t} (\cos ks_1d - \cos ks_2d) \sin ktd + (s_1 \sin ks_1d - s_2 \sin ks_2d) \cos ktd - s_1SC + s_2CS \right];$$

$$k_{21} = -k_{54} = -k [(r_1 + r_2SS - r_1CC) \sin ktd + (r_3SC + r_4CS) \cos ktd - r_3 \sin ks_1d - r_4 \sin ks_2d];$$

$$r_1 = A_2A_3(E_1 + E_2); \quad r_2 = -(2A_1^2G^* + A_2^2E_2 + A_3^2E_1); \quad r_3 = A_1A_2(E_2 + 2G^*); \quad r_4 = -A_1A_3(E_1 + 2G^*);$$

$$k_{22} = k_{55} = k [(r_1 + r_2SS + r_3CC) \cos ktd + (r_4SC + r_5CS) \sin ktd + r_6 \cos ks_1d + r_7 \cos ks_2d];$$

$$r_1 = s_2(D_1 - D_3)(D_2 - 1)A_4 + s_1(D_2 - D_3)(D_1 - 1)A_5; \quad r_2 = -A_2A_4(D_1 - 1) - A_3A_5(D_2 - 1);$$

$$r_3 = A_3A_4(1 - D_1) - A_1A_6 - A_2A_5(D_2 - 1); \quad r_4 = A_1A_4(1 - D_1) - A_3A_6; \quad r_5 = A_1A_5(D_2 - 1) + A_2A_6;$$

$$r_6 = A_1(D_3 - 1)(E_1 - 2G^*) + s_2tA_2(E_1 - E_2)(D_2 - 1); \quad r_7 = A_1(1 - D_3)(E_2 - 2G^*) - s_1tA_3(E_1 - E_2)(D_1 - 1);$$

$$k_{23} = k/n[(r_1 + r_2SS + r_3CC) \cos ktd + (r_4SC + r_5CS) \sin ktd + r_6 \cos ks_1d + r_7 \cos ks_2d];$$

$$r_1 = s_1t(2G^* - E_1)A_3 - s_2t(E_2 - 2G^*)A_2; \quad r_2 = A_2A_4 + A_3A_5; \quad r_3 = A_1(E_1 - E_2) + A_2A_5 + A_3A_4;$$

$$r_4 = A_1A_4 + A_3(E_1 - E_2); \quad r_5 = -A_1A_5 - A_2(E_1 - E_2); \quad r_6 = A_1(2G^* - E_1) + s_2tA_2(E_2 - E_1);$$

$$r_7 = A_1(E_2 - 2G^*) + s_1tA_3(E_1 - E_2);$$

$$k_{24} = -k[(r_1 \cos ks_1d - r_1 \cos ks_2d) \sin ktd + (r_2 \sin ks_1d + r_3 \sin ks_2d) \cos ktd - r_2SC - r_3CS];$$

$$r_1 = A_2A_3(E_1 - E_2); \quad r_2 = A_1A_2(E_2 - 2G^*); \quad r_3 = -A_1A_3(E_1 - 2G^*);$$

$$k_{25} = k_{32} = -k[(r_1 \sin ks_1d + r_2 \sin ks_2d) \sin ktd + (r_3 \cos ks_1d + r_4 \cos ks_2d) \cos ktd + r_5SS + r_6CC + r_7];$$

$$r_1 = -A_1A_4(D_1 - 1) - A_3A_6; \quad r_3 = A_1(D_3 - 1)(E_2 - 2G^*) + s_1tA_3(E_1 - E_2)(D_1 - 1);$$

$$r_2 = A_2A_6 + A_1A_5(D_2 - 1); \quad r_4 = -A_1(D_3 - 1)(E_1 - 2G^*) - s_2tA_2(E_1 - E_2)(D_2 - 1);$$

$$r_5 = -A_2A_4(D_1 - 1) - A_3A_5(D_2 - 1); \quad r_6 = -s_1A_5(D_1 - 1)(D_2 - D_3) - s_2A_4(D_1 - D_3)(D_2 - 1);$$

$$r_7 = A_1A_6 + A_2A_5(D_2 - 1) + A_3A_4(D_1 - 1);$$

$$k_{26} = -k/n[(r_1 \sin ks_1d + r_2 \sin ks_2d) \sin ktd + (r_3 \cos ks_1d + r_4 \cos ks_2d) \cos ktd + r_5SS + r_6CC + r_7];$$

$$r_1 = A_1A_4 + A_3(E_1 - E_2); \quad r_2 = -A_1A_5 - A_2(E_1 - E_2); \quad r_3 = s_1tA_3(E_2 - E_1) - s_2A_4(D_1 - D_2);$$

$$r_4 = A_1(E_1 - 2G^*) + s_2tA_2(E_1 - E_2); \quad r_5 = A_2A_4 + A_3A_5;$$

$$r_6 = s_1A_5(D_2 - D_3) + s_2A_4(D_1 - D_3); \quad r_7 = -A_1(E_1 - E_2) - A_2A_5 - A_3A_4;$$

$$k_{31} = -k_{64} = -k^{-1}[(r_1 + r_2SS - r_1CC) \sin ktd + (r_3SC + r_4CS) \cos ktd - r_3 \sin ks_1d - r_4 \sin ks_2d];$$

$$r_1 = -A_2A_3(P_1 + P_2); \quad r_2 = A_3^2P_1 + A_2^2P_2; \quad r_3 = -A_1A_2P_2; \quad r_4 = A_1A_3P_1;$$

$$k_{32} = k^{-1}[(r_1 + r_2SS + r_3CC) \cos ktd + (r_4SC + r_5CS) \sin ktd + r_6 \cos ks_1d + r_7 \cos ks_2d];$$

$$\begin{aligned}
r_1 &= -P_1 s_1 t (D_1 - 1) A_3 - P_2 s_2 t (D_2 - 1) A_2; & r_2 &= A_3 B_4 + A_2 B_5; & r_3 &= A_1 B_6 + A_2 B_4 + A_3 B_5; \\
r_4 &= A_1 B_5 + A_3 B_6; & r_5 &= -A_1 B_4 - A_2 B_6; & r_6 &= -P_1 A_1 (D_3 - 1) - s_2 t A_2 (P_1 - P_2) (D_2 - 1); \\
r_7 &= P_2 A_1 (D_3 - 1) + s_1 t A_3 (P_1 - P_2) (D_1 - 1);
\end{aligned}$$

$$k_{33} = k_{66} = (nk)^{-1} [(r_1 + r_2 SS + r_3 CC) \cos ktd + (r_4 SC + r_5 CS) \sin ktd + r_6 \cos ks_1 d + r_7 \cos ks_2 d];$$

$$\begin{aligned}
r_1 &= P_1 s_1 t A_3 + P_2 s_2 t A_2; & r_2 &= -P_2 s_1 t A_2 - P_1 s_2 t A_3; & r_4 &= (P_2 - P_1) A_3 - P_2 s_1 t A_4; & r_5 &= (P_1 - P_2) A_2 + P_1 s_2 t A_4; \\
r_3 &= -P_1 s_2 t A_2 - P_2 s_1 t A_3 - (P_1 - P_2) A_1; & r_6 &= P_1 A_1 + s_2 t A_2 (P_1 - P_2); & r_7 &= s_1 t A_3 (P_2 - P_1) - P_2 A_1;
\end{aligned}$$

$$k_{34} = k^{-1} [(r_1 \cos ks_1 d - r_1 \cos ks_2 d) \sin ktd + (r_2 \sin ks_1 d + r_3 \sin ks_2 d) \cos ktd - r_2 SC - r_3 CS];$$

$$r_1 = A_2 A_3 (P_1 - P_2); \quad r_2 = A_1 A_2 P_2; \quad r_3 = -A_1 A_3 P_1;$$

$$k_{35} = k^{-1} [(r_1 \sin ks_1 d + r_2 \sin ks_2 d) \sin ktd + (r_3 \cos ks_1 d + r_4 \cos ks_2 d) \cos ktd + r_5 SS + r_6 CC + r_7];$$

$$\begin{aligned}
r_1 &= -A_1 B_5 - A_3 B_6; & r_2 &= A_1 B_4 + A_2 B_6; & r_3 &= A_1 P_2 (D_3 - 1) + s_1 t A_3 (P_1 - P_2) (D_1 - 1); \\
r_4 &= -A_1 P_1 (D_3 - 1) - s_2 t A_2 (P_1 - P_2) (D_2 - 1); & r_5 &= -A_2 B_5 - A_3 B_4; \\
r_6 &= -P_1 s_1 t A_3 (D_1 - 1) - P_2 s_2 t A_2 (D_2 - 1); & r_7 &= A_1 B_6 + A_2 B_4 + A_3 B_5;
\end{aligned}$$

$$k_{36} = k_{63} = (nk)^{-1} [(r_1 \sin ks_1 d + r_2 \sin ks_2 d) \sin ktd + (r_3 \cos ks_1 d + r_4 \cos ks_2 d) \cos ktd + r_5 SS + r_6 CC + r_7];$$

$$\begin{aligned}
r_1 &= s_1 t A_1 P_2 + A_3 (P_1 - P_2); & r_2 &= -s_2 t A_1 P_1 - A_2 (P_1 - P_2); & r_3 &= -P_2 A_1 - s_1 t A_3 (P_1 - P_2); \\
r_4 &= P_1 A_1 + s_2 t A_2 (P_1 - P_2); & r_5 &= t (s_2 P_1 A_3 + s_1 P_2 A_2); & r_6 &= t (s_1 P_1 A_3 + s_2 P_2 A_2); \\
r_7 &= -A_1 (P_1 - P_2) - s_2 t A_2 P_1 - s_1 t A_3 P_2;
\end{aligned}$$

$$k_{42} = -k(1+t^2) A_1 G^* [(r_1 \cos ks_1 d - r_1 \cos ks_2 d) \sin ktd + (r_2 \sin ks_1 d + r_3 \sin ks_2 d) \cos ktd - r_2 SC - r_3 CS];$$

$$r_1 = -(D_3 - 1)/t; \quad r_2 = -s_1 (D_1 - 1); \quad r_3 = s_2 (D_2 - 1);$$

$$k_{43} = -n^{-1} k(1+t^2) A_1 G^* [t^{-1} (\cos ks_1 d - \cos ks_2 d) \sin ktd + (s_1 \sin ks_1 d - s_2 \sin ks_2 d) \cos ktd - s_1 SC + s_2 CS];$$

$$k_{51} = -k [(r_1 \cos ks_1 d - r_1 \cos ks_2 d) \sin ktd + (r_2 \sin ks_1 d + r_3 \sin ks_2 d) \cos ktd - r_2 SC - r_3 CS];$$

$$r_1 = -A_2 A_3 (E_1 - E_2); \quad r_2 = -A_1 A_2 (E_2 - 2G^*); \quad r_3 = A_1 A_3 (E_1 - 2G^*);$$

$$k_{53} = n^{-1} k [(r_1 \sin ks_1 d + r_2 \sin ks_2 d) \sin ktd + (r_3 \cos ks_1 d + r_4 \cos ks_2 d) \cos ktd + r_5 SS + r_6 CC + r_7];$$

$$r_1 = -A_1 A_4 - A_3 (E_1 - E_2); \quad r_2 = A_1 A_5 + A_2 (E_1 - E_2); \quad r_3 = A_1 (E_2 - 2G^*) + s_1 t A_3 (E_1 - E_2);$$

$$r_4 = -A_1 (E_1 - 2G^*) - s_2 t A_2 (E_1 - E_2); \quad r_5 = -(A_2 A_4 + A_3 A_5);$$

$$r_6 = -A_4s_2(D_1 - D_3) - A_5s_1(D_2 - D_3); \quad r_7 = A_1(E_1 - E_2) + A_2A_5 + A_3A_4$$

$$k_{56} = n^{-1}k[(r_1 + r_2SS + r_3CC) \cos ktd + (r_4SC + r_5CS) \sin ktd + r_6 \cos ks_1d + r_7 \cos ks_2d];$$

$$r_1 = -s_2A_4(D_1 - D_3) - s_1A_5(D_2 - D_3); \quad r_2 = A_2A_4 + A_3A_5; \quad r_3 = A_1(E_1 - E_2) + A_2A_5 + A_3A_4;$$

$$r_4 = A_1A_4 + A_3(E_1 - E_2); \quad r_5 = -A_1A_5 - A_2(E_1 - E_2);$$

$$r_6 = -A_1(E_1 - 2G^*) - s_2tA_2(E_1 - E_2); \quad r_7 = A_1(E_2 - 2G^*) + s_1tA_3(E_1 - E_2);$$

$$k_{61} = k^{-1}[(r_1 \cos ks_1d - r_1 \cos ks_2d) \sin ktd + (r_2 \sin ks_1d + r_3 \sin ks_2d) \cos ktd - r_2SC - r_3CS];$$

$$r_1 = -A_2A_3(P_1 - P_2); \quad r_2 = -A_1A_2P_2; \quad r_3 = A_1A_3P_1;$$

$$k_{62} = -k^{-1}[(r_1 \sin ks_1d + r_2 \sin ks_2d) \sin ktd + (r_3 \cos ks_1d + r_4 \cos ks_2d) \cos ktd + r_5SS + r_6CC + r_7];$$

$$r_1 = A_1B_5 + A_3B_6; \quad r_2 = -A_1B_4 - A_2B_6; \quad r_3 = -P_2A_1(D_3 - 1) - A_3B_5(P_1/P_2 - 1);$$

$$r_4 = P_1A_1(D_3 - 1) + s_2tA_2(P_1 - P_2)(D_2 - 1); \quad r_5 = A_2B_5 + A_3B_4;$$

$$r_6 = A_2B_4P_2/P_1 + A_3B_5P_1/P_2; \quad r_7 = -A_1B_6 - A_2B_4 - A_3B_5;$$

$$k_{65} = k^{-1}[(r_1 + r_2SS + r_3CC) \cos ktd + (r_4SC + r_5CS) \sin ktd + r_6 \cos ks_1d + r_7 \cos ks_2d];$$

$$r_1 = -A_2B_4P_2/P_1 - A_3B_5P_1/P_2; \quad r_2 = A_2B_5 + A_3B_4; \quad r_3 = A_1B_6 + A_2B_4 + A_3B_5;$$

$$r_4 = A_1B_5 + A_3B_6; \quad r_5 = -A_1B_4 - A_2B_6; \quad r_6 = -P_1A_1(D_3 - 1) + A_2B_4(P_2/P_1 - 1);$$

$$r_7 = P_2A_1(D_3 - 1) + A_3B_5(P_1/P_2 - 1);$$

2. The elements r_{ij} and r_0 of dynamic-stiffness matrix of half-space $\mathbf{S}_{P1-P2-SV}^R$.

$$r_0 = s_1(D_1 - D_3) - s_2(D_2 - D_3) + ts_1s_2(D_1 - D_2); \quad r_{11} = iG^*ks_1s_2(1 + t^2)(D_1 - D_2);$$

$$r_{12} = G^*k[(t^2 - D_3 + 1)(s_1 - s_2) + s_1(D_1 - D_3) - s_2(D_2 - D_3) + 2ts_1s_2(D_1 - D_2) - t^2(s_1D_1 - s_2D_2)];$$

$$r_{13} = kG^*(1 + t^2)(s_1 - s_2)/n; \quad r_{21} = -k[E_1s_2(D_2 - D_3) - E_2s_1(D_1 - D_3) - 2G^*ts_1s_2(D_1 - D_2)];$$

$$r_{22} = ik[(E_1 - E_2)(D_3 - 1) + ts_2(D_2 - 1)(E_1 - 2G^*) - ts_1(D_1 - 1)(E_2 - 2G^*)];$$

$$r_{23} = -ik[ts_2(E_1 - 2G^*) - ts_1(E_2 - 2G^*) + E_1 - E_2]/n; \quad r_{31} = [P_1s_2(D_2 - D_3) - P_2s_1(D_1 - D_3)]/k;$$

$$r_{32} = -i[(P_1 - P_2)(D_3 - 1) + P_1ts_2(D_2 - 1) - P_2ts_1(D_1 - 1)]/k; \quad r_{33} = i[P_1(1 + ts_2) - P_2(1 + ts_1)]/kn.$$